Influence of temperature dependent inelastic scattering on the superconducting proximity effect

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We have measured the differential resistance of mesoscopic gold wires of different lengths connected to an aluminum superconductor as a function of temperature and voltage. Our experimental results differ substantially from theoretical predictions which assume an infinite temperature independent gap in the superconductor. In addition to taking into account the temperature dependence of the gap, we must also introduce a temperature dependent inelastic scattering length in order to fit our data.

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When a superconductor (S) is brought into intimate electrical contact with a normal metal (N) wire, induced pair correlations caused by Andreev reflection [1] result in an enhancement of the normal metal diffusion constant D_N [2]. According to the quasiclassical theory of superconductivity, this enhancement has a non-monotonic dependence on the energy ε of the quasiparticles in the normal metal, with a maximum at a characteristic energy scale $E_c = \hbar D_N/L^2$, where L is the length of the wire [3,4]. As a consequence, the differential resistance of the wire is predicted to have a non-monotonic dependence as a function of temperature or voltage, with minimum values at $T_{min} \sim E_c/k_B$ and $V_{min} \sim E_c/e$ respectively.

A number of groups have reported observing this reentrant behavior in the differential resistance of normal one-dimensional wires connected to a superconductor. Charlat *et al.* [5] measured the conductance of a Cu loop connected to a superconducting Al reservoir as a function of temperature, and found good agreement with quasiclassical predictions [3]. Similar results were obtained by Petrashov *et al.* [6] when they measured magnetoresistance oscillations in Pb-Ag structures. However, in other experiments [7–10], T_{min} or V_{min} did not correspond to the values predicted by theory. Furthermore, even in a single sample, the voltage and temperature scales at which the minima in resistance were observed did not agree ($eV_{min} \ll k_B T_{min}$) [10].

In all these experiments, many simplifying assumptions were made to make the calculations tractable. For example, it was typically assumed that the gap Δ in the superconductor was temperature independent and much larger than the energy ε of the quasiparticles, an assumption which is clearly not valid in the experimental regime of interest for those experiments which used Al as the superconductor. More recently, some groups have attempted to improve the agreement between theory and experiment by taking these factors into account [11], but the results are still not satisfactory, especially near the superconducting transition temperature T_c of the superconductor.

In order to clarify these issues and provide a quantitative test of the quasiclassical theory of the proximity effect in mesoscopic samples, we set out to measure the length dependence of the proximity effect in diffusive Au wires. The samples were designed to correspond to the simplest geometry analyzed theoretically by Nazarov and Stoof [4]: a single normal Au wire connected on one end to a superconducting Al reservoir, and to a normal Au reservoir on the other end. For wires in which the electron phase coherence length L_ϕ is longer than the length L of the wire (as is the case in our samples), Nazarov and Stoof predict that $V_{min} \sim 5E_c/e$ and $T_{min} \sim 5E_c/k_B$. However, in our samples, we find that the situation is much more complicated. T_{min} is approximately a factor of 5 less than predicted by Nazarov and Stoof. In addition, we find that V_{min} is typically much less than $k_B T_{min}/e$ in the same sample due to heating of the quasiparticles in the wires by the dc bias, in spite of the fact that L is much smaller than L_ϕ. In order to fit our data within the framework of the quasiclassical theory, we need to take into account the temperature dependence of Δ, as well as the temperature dependence of the inelastic scattering length of the quasiparticles in the normal metal.

The Au/Al samples for this experiment were fabricated by standard electron-beam lithography techniques onto oxidized Si substrates. Figure 1 shows a scanning electron micrograph of one of our samples. In consists of five Au wires of length L ranging from $\sim$ 0.75-1.5 μm. Each wire is connected to a Au reservoir on one end and a large superconducting Al reservoir on the other. In order to minimize variations in the transparency of the NS interface barriers, all wires are connected to the same Al reservoir. The Au layer was patterned and deposited first; following a second level of lithography, the Al film was deposited immediately after cleaning the Au layer with an oxygen plasma etch. This cleaning step guaranteed a transparent NS interface, as evidenced by a consistent interfacial resistance of less than 0.1 Ω. Unlike most previous experiments, each Au wire also has two voltage probes in order to enable us to make four-terminal differential resistance measurements of the proximity wire

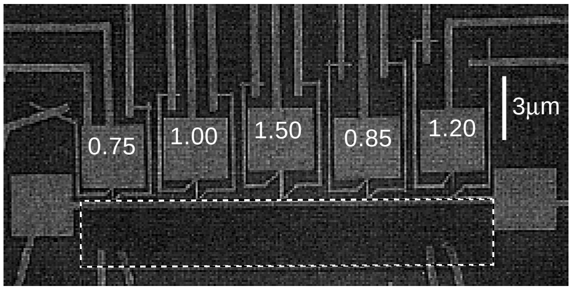

FIG. 1. Scanning electron micrograph of our device structure. The light colored areas are 50 nm thick Au, and the dotted line denotes the edge of the 80 nm thick Al contact.

alone with no direct contribution from the Al reservoir.

The samples were measured in a dilution refrigerator using standard four-terminal techniques, with ac currents small enough to avoid self-heating. A total of six sample sets were measured: three in the geometry of Fig. 1, and three in a similar but slightly different geometry, with the length of the wires ranging from 0.5-3.0 μm. We present here data from three wires in the set shown in Fig. 1; data from wires in other sets showed similar behavior. For this set, other relevant sample parameters are as follows: Au film thickness 50 nm, Au wire width 120 nm, Al film thickness 80 nm, Au diffusion constant $D_N = 300$ cm^2/sec, Au thermal diffusion length $L_T = \sqrt{\hbar D/k_B T} = 0.48$ μm at 1 K, and Al superconducting transition temperature $T_c = 1.20$ K. The electron phase coherence length was estimated to be 3.7 μm at 27 mK from weak localization measurements on a co-evaporated Au wire.

Figure 2(a) shows the zero-bias differential resistance $R(T)$ of three Au wires of length L=1.02, 1.23 and 1.47 μm from a single set normalized to their normal state values R_N as a function of T [12]. All three wires show reentrant behavior, with the minimum in resistance occuring at a lower temperature T_{min} for the longer wires. It is immediately apparent that T_{min} does not scale as $1/L^2$. Furthermore, the observed value of T_{min} is much smaller than the value of $5E_c/k_B$ predicted by Nazarov and Stoof. For example, for the L=1.47 μm wire, $5E_c/k_B$=0.534 K, whereas $T_{min} \sim 0.148$ K. In the case of the dc voltage bias dependence, which is shown in Fig. 2(b), the situation is even more complicated. V_{min} does not show any systematic dependence on L: V_{min} for the 1.02 μm and 1.47 μm wires are approximately the same, while V_{min} for the 1.23 μm wire has a smaller value. For all wires, V_{min} is again much less than the theoretical prediction of $5E_c/e$.

The non-systematic dependence of V_{min} on L is due to the increase in the effective temperature of the quasiparticles in the Au wires due to heating by the dc bias [14]. This can be shown directly by measuring the differential resistance of one Au wire as a function of dc voltage bias while simultaneously measuring the differential resistance of a second Au wire adjacent to the first. The

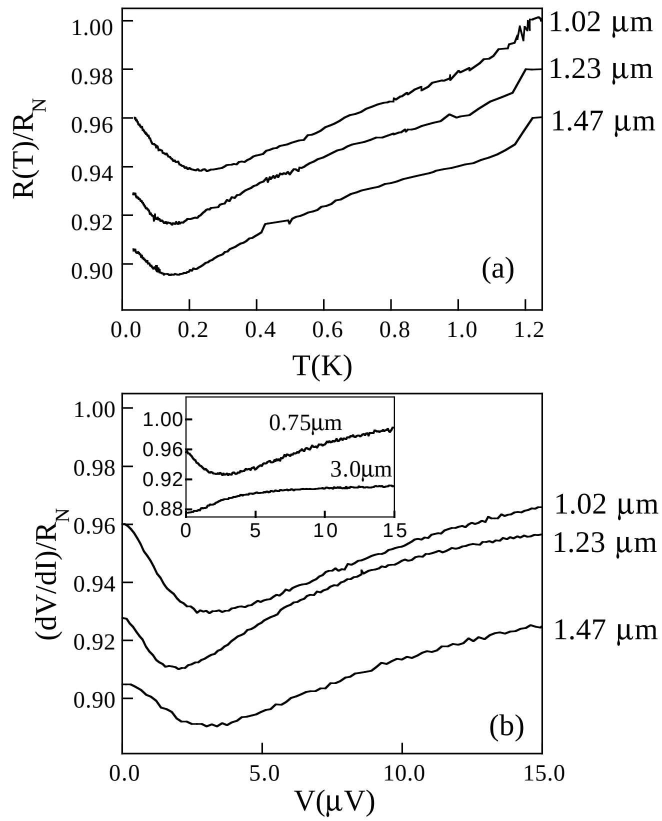

FIG. 2. (a) Normalized differential resistance measured as a function of temperature for wires of length L = 1.02, 1.23, and 1.47 μm with normal state resistances R_N = 2.41, 2.46, and 3.69 Ω respectively. (b) Normalized differential resistance measured as a function of voltage for the same wires. The 1.23 and 1.47 μm curves in (a) and (b) have been offset by -0.02 and -0.04 Ω for clarity. Inset: Normalized differential resistance for a 0.75 μm wire (upper) as a function of dc voltage bias at T= 29 mK: the simultaneous measurement of an adjacent 3.00 μm wire (lower) acts as a thermometer.

inset to Fig. 2(b) shows the differential resistance $R(V) = dV/dI$ of a L=0.75 μm Au wire from a second sample set as a function of dc bias V, along with the resistance of the 3.0 μm wire immediately adjacent to it. In the absence of heating effects, one would not expect the resistance to change as a function of V across the 0.75 μm wire, since the 3.0 μm wire has no dc current flowing through it. By correlating $R(T)$ of the 3.0 μm wire with its change in resistance as a function of V across the 0.75 μm wire, we can estimate the effective temperature increase as a function of V. For example, a bias of $V = 5$ μV corresponds to an increase in the electron temperature in the 3.0 μm wire to 250 mK from 29 mK. Since V_{min} is expected to decrease as T increases, this increase in the effective temperature would lead to a decrease in the apparent value of V_{min}, as we observe. In principle, it should be possible to calculate and correct for this effect by taking into account the mechanisms for heat generation and dissipation in the sample [13]; in practice, our

geometry is complex enough to make this very difficult. Consequently, in the remainder of the paper, we shall confine ourselves to the discussion of the temperature dependent resistance $R(T)$. We should note, however, that our sample design, with its large normal and superconducting reservoirs, might be expected to minimize heating due to the dc bias; in samples without such reservoirs (as in many previous experiments), the problem will be more acute.

We now come to the discussion of the temperature dependent resistance. The procedure for calculating the normalized resistance $R(T)/R_N$ from the quasiclassical theory of superconductivity has been discussed by many authors [3], and we shall not repeat it in detail here. Briefly, one needs to solve the Usadel equation [15]

$$\hbar D_N \frac{\partial^2\theta(\varepsilon,x)}{\partial x^2} + 2i\varepsilon\sin\theta(\varepsilon,x) = 0 \qquad (1)$$

for the complex angle θ which parameterizes the nonequilibrium superconducting Green's functions. This equation is solved subject to the boundary condition that

$$\theta(\varepsilon) = \begin{cases} \frac{\pi}{2} + i\frac{1}{2}\ln\frac{\Delta(T)+\varepsilon}{\Delta(T)-\varepsilon} & \text{if } \varepsilon < \Delta(T) \\ i\frac{1}{2}\ln\frac{\varepsilon+\Delta(T)}{\varepsilon-\Delta(T)} & \text{if } \varepsilon > \Delta(T) \end{cases} \qquad (2)$$

at the superconducting reservoir ($x = 0$), and $\theta = 0$ at the normal reservoir ($x = L$). The enhanced diffusion coefficient is then obtained by integrating over the length of the sample

$$D(\varepsilon) = \frac{D_N}{1/L\int_0^L dx\,\mathrm{sech}^2\left[\Im\theta(\varepsilon,x)\right]}. \qquad (3)$$

Finally, $R(T)/R_N$ is obtained by convoluting $D(\varepsilon)$ with a thermal kernel

$$\frac{R(T)}{R_N} = \left[\int_0^\infty \frac{D(\varepsilon)}{2k_BT\cosh^2\left(\frac{\varepsilon}{2k_BT}\right)}\right]^{-1}. \qquad (4)$$

In their calculation for this geometry, Nazarov and Stoof [4] assume that $\varepsilon \ll \Delta(T)$ at all temperatures T; under this assumption, the boundary conditions Eq.(2) at the superconducting reservoir reduce simply to $\theta = \pi/2$. A better approximation can be made by taking into account the temperature dependence of the gap. Recently, Petrashov *et al.* [11] fit $R(T)$ for their Ag/Al Andreev interferometers to the quasiclassical theory by using $\Delta(T)$ as a fitting parameter. The resulting temperature dependence of Δ they obtained was in agreement with the predictions of the microscopic theory of Bardeen, Cooper and Schrieffer (BCS) [16]; however, the zero temperature value $\Delta(0)$ was a factor of five smaller than would be expected from T_c of the Al film.

Almost all analyses of the mesoscopic proximity effect neglect inelastic scattering of the quasiparticles, assuming that the inelastic scattering length L_{in} is much longer than the sample dimensions in the temperature range of interest [17]. If $L_{in} < L$, L_{in} determines the effective sample length. Clearly, if L_{in} changes as a function of temperature, this will affect the measured $R(T)/R_N$. It is instructive to investigate the effect of temperature dependent inelastic scattering on the proximity effect. This can be accomplished by introducing an imaginary component $\gamma(T)$ to the energy of the quasiparticles [18], $\varepsilon \to \varepsilon + i\gamma(T)$, with a corresponding length $L_\gamma = \sqrt{\hbar D/2\gamma}$. Figure 3(a) shows the experimental $R(T)/R_N$ curves for the three wires of Fig. 2, along with fits to the quasiclassical theory using L_γ as a temperature dependent fitting parameter. In order to obtain these fits, we assumed a BCS-like temperature dependence of the gap, with $\Delta(0)$=182 μeV estimated from T_c of the Al film. Below T_{min}, the fits become insensitive to the choice of γ, due to the fact that L_γ becomes longer than the length of the wires. Consequently, we have only fit the data down to T_{min} [19]. For comparison, Fig. 3(b) shows curves calculated assuming a temperature dependent gap with $\Delta(0) = 0.2 \times 182$ μeV, but a constant

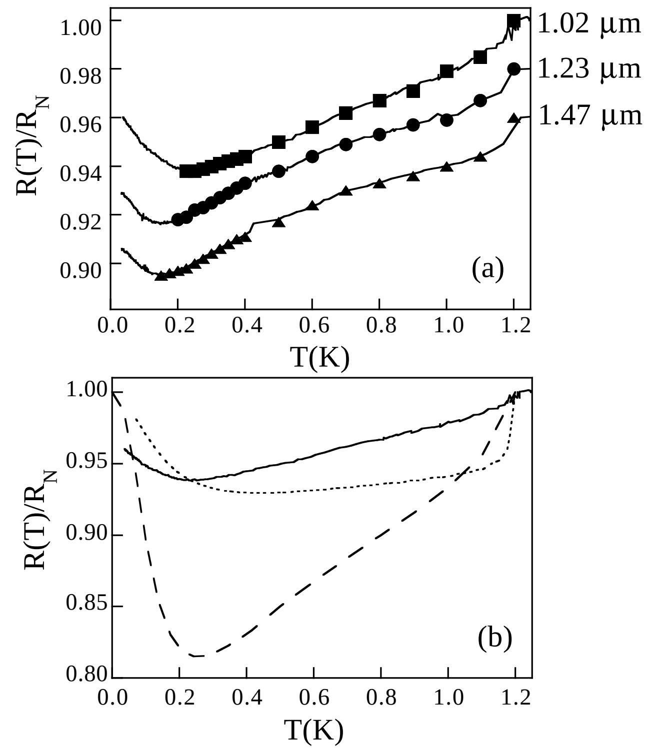

FIG. 3. (a) Solid lines: data from Fig. 2(a); solid symbols, simulation of $R(T)/R_N$ with a temperature dependence fitting parameter L_γ, and a temperature dependent gap, as described in the text. The 1.23 and 1.47 μm curves here and in (b) have been offset by -0.02 and -0.04 Ω for clarity. (b) Solid line: data for 1.0 μm wire from Fig. 2(a); dashed line, simulation of $R(T)/R_N$ for its temperature dependence using the recipe of Petrashov *et al.*[11], with $L_\gamma = 2.0$ μm and $\Delta(0) = \beta\times$ 182 μeV with $\beta = 0.2$, the same value as used by Petrashov *et al.*

temperature independent $L_\gamma = 2.0$ μm, as in Petrashov *et al.* [11]. These curves deviate from the experimental results at almost all temperatures [20].

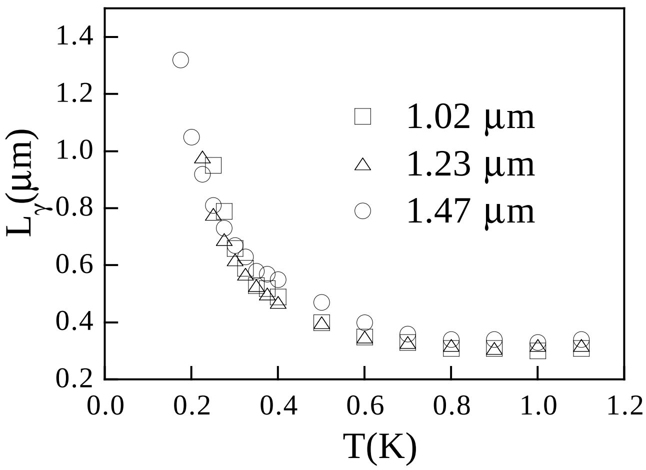

FIG. 4. $L_\gamma(T)$ obtained by fitting to the data in Fig. 2(a) for the $L = 1.02$, 1.23, and 1.47 μm wires. Even though the fits were performed independently from one another, the calculated values lie on the same curve.

Figure 4 shows L_γ obtained from these fits obtained as described above as a function of T for the 1.02, 1.24 and 1.47 μm long wires. Even though the fits were performed independently, the resulting $L_\gamma(T)$ for each wire is essentially the same. L_γ saturates at higher temperatures at ~ 0.3 μm, and appears to diverge at lower temperatures. The values of L_γ obtained are much smaller than the experimentally determined value for $L_\phi \sim 3.7$ μm. In addition, the temperature dependence of L_γ is much sharper than the expected temperature dependence of L_ϕ [21,22]. Consequently, it appears that L_γ is not directly related to the phase coherence length in the normal metal.

Recently, Giroud *et al.* [23] observed a reentrant proximity effect in a Co loop connected to an Al island, even though L_ϕ in the Co film was short, as evidenced by the absence of quantum interference effects. Our work is another indication that L_ϕ may not be the most relevant length scale for the proximity effect.

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